

Exercise 4-2

$$\begin{aligned} \lim_{+\infty} \frac{\sqrt{x^2 - 3x + 1}}{x + 2} &= \lim_{+\infty} \frac{\sqrt{x^2 \left(1 - \frac{3}{x} + \frac{1}{x^2}\right)}}{x \left(1 + \frac{2}{x}\right)} \\ &= \lim_{+\infty} \frac{x \sqrt{1 - \frac{3}{x} + \frac{1}{x^2}}}{x \left(1 + \frac{2}{x}\right)} = \frac{1}{1} = 1 \end{aligned}$$

$$\lim_{+\infty} \frac{\sqrt{x + 2}}{x - 1} = \lim_{+\infty} \frac{\sqrt{x^2 \left(\frac{1}{x} + \frac{2}{x^2}\right)}}{x \left(1 - \frac{1}{x}\right)} = \lim_{+\infty} \frac{\sqrt{\frac{1}{x} + \frac{2}{x^2}}}{1 - \frac{1}{x}} = 0$$

$$\begin{aligned} \lim_{+\infty} \frac{\sqrt{x^2 - 2x} - 2x}{x} &= \lim_{+\infty} \frac{\sqrt{x^2 \left(1 - \frac{2}{x}\right)} - 2x}{x} \\ &= \lim_{+\infty} \frac{x \left(\sqrt{1 - \frac{2}{x}} - 2\right)}{x} = -1 \end{aligned}$$

$$\lim_{-\infty} \sqrt{x^2 + 3x - 5} - 2x = "+\infty + \infty" = +\infty$$

$$\lim_{-\infty} x^2 + 3x - 5 = \lim_{-\infty} x^2 = +\infty \quad \rightarrow \infty$$

$$\begin{aligned} \lim_{+\infty} \sqrt{x^2 + 3x - 5} - 2x &= \lim_{+\infty} x \sqrt{1 + \frac{3}{x} - \frac{5}{x^2}} - 2x \\ &= \lim_{+\infty} x \left(\sqrt{1 + \frac{3}{x} - \frac{5}{x^2}} - 2\right) \\ &= "+\infty (-1)" = -\infty \end{aligned}$$

Exercice 4-3

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} \sqrt{x^2 - 3x + 1} - x &= \lim_{x \rightarrow +\infty} x \sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} - x \\
 &= \lim_{x \rightarrow +\infty} x \left(\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} - 1 \right) \\
 &= \lim_{x \rightarrow +\infty} x \left(\frac{1 - \frac{3}{x} + \frac{1}{x^2} - 1}{\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} + 1} \right) \\
 &= \lim_{x \rightarrow +\infty} \frac{-3 + \frac{1}{x}}{\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} + 1} = -\frac{3}{2}
 \end{aligned}$$

Exercice 4-4

$$\begin{aligned}
 \lim_{x \rightarrow +\infty} \sqrt{4x^2 - 2x + 1} - 2x &= \lim_{x \rightarrow +\infty} \sqrt{x^2 \cdot \left(4 - \frac{2}{x} + \frac{1}{x^2} \right)} - 2x \\
 &= \lim_{x \rightarrow +\infty} x \cdot \sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} - 2x \\
 &= \lim_{x \rightarrow +\infty} x \cdot \left(\sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} - 2 \right) \\
 &= \lim_{x \rightarrow +\infty} x \cdot \left(\frac{4 - \frac{2}{x} + \frac{1}{x^2} - 4}{\sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} + 2} \right) \\
 &= \lim_{x \rightarrow +\infty} \frac{-2 + \frac{1}{x}}{\sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} + 2} = -\frac{2}{4} \\
 &= -\frac{1}{2}
 \end{aligned}$$

Exercice 4-5

$$\begin{aligned}\lim_{-\infty} \sqrt{x^2 + 3x + 7} + x &= \lim_{-\infty} -x \sqrt{1 + \frac{3}{x} + \frac{7}{x^2}} + x \\ &= \lim_{-\infty} -x \left(\sqrt{1 + \frac{3}{x} + \frac{7}{x^2}} - 1 \right) \\ &= \lim_{-\infty} -x \left(\frac{1 + \frac{3}{x} + \frac{7}{x^2} - 1}{\sqrt{1 + \frac{3}{x} + \frac{7}{x^2}} + 1} \right) \\ &= \lim_{-\infty} \frac{-3 - \frac{7}{x}}{\sqrt{1 + \frac{3}{x} + \frac{7}{x^2}} + 1} = \frac{-3}{2}\end{aligned}$$

Exercice 4-6

$$\begin{aligned}\lim_{+\infty} x - 2\sqrt{x} + 9 &= \lim_{+\infty} x \left(1 - 2 \frac{\sqrt{x}}{x} + \frac{9}{x} \right) \\ &= \lim_{+\infty} x \left(1 - 2 \left(\frac{1}{\sqrt{x}} \right) + \frac{9}{x} \right) = +\infty\end{aligned}$$

$$\begin{aligned}\lim_{+\infty} \frac{\sqrt{x}}{x+1} &= \lim_{+\infty} \frac{x \left(\frac{\sqrt{x}}{x} \right)}{x \left(1 + \frac{1}{x} \right)} = \frac{\frac{1}{\sqrt{x}}}{1 + \frac{1}{x}} = \frac{0}{1} \\ &= 0\end{aligned}$$

$$\begin{aligned}\lim_{+\infty} \frac{x - 2\sqrt{x} + 2}{x} &= \lim_{+\infty} 1 - 2 \frac{\sqrt{x}}{x} + \frac{2}{x} \\ &= \lim_{+\infty} 1 - 2 \left(\frac{1}{\sqrt{x}} \right) + \frac{2}{x} = 1\end{aligned}$$

$$\lim_{+\infty} \frac{x + \sqrt{x}}{x - \sqrt{x}} = \lim_{+\infty} \frac{x \left(1 + \frac{\sqrt{x}}{x} \right)}{x \left(1 - \frac{\sqrt{x}}{x} \right)} = \lim_{+\infty} \frac{1 + \frac{1}{\sqrt{x}}}{1 - \frac{1}{\sqrt{x}}} = 1$$

$$\begin{aligned}\lim_{+\infty} \frac{x+2}{\sqrt{x}} &= \lim_{+\infty} \frac{x}{\sqrt{x}} + \frac{2}{\sqrt{x}} \\ &= \lim_{+\infty} \sqrt{x} + \frac{2}{\sqrt{x}} = +\infty\end{aligned}$$