Exercice 4-2

$$\lim_{+\infty} \frac{\sqrt{x^2-3x+1}}{x+2} = \lim_{+\infty} \frac{\sqrt{x^2\left(1-\frac{3}{x}+\frac{1}{x^2}\right)}}{x\left(1+\frac{2}{x}\right)}$$

$$= \lim_{+\infty} \frac{x\sqrt{1-\frac{3}{x}+\frac{1}{x^2}}}{x\left(1+\frac{2}{x}\right)} = \frac{1}{1} = 1$$

$$\lim_{+\infty} \frac{\sqrt{x+2}}{x-1} = \lim_{+\infty} \frac{\sqrt{x^2\left(\frac{1}{x}+\frac{2}{x^2}\right)}}{x\left(1-\frac{1}{x}\right)} = \lim_{+\infty} \frac{\sqrt{\frac{1}{x}+\frac{2}{x^2}}}{1-1/x} = 0$$

$$\lim_{+\infty} \frac{\sqrt{x^2-2x}-2x}{x} = \lim_{+\infty} \frac{\sqrt{x^2\left(1-\frac{2}{x}\right)}-2x}{x}$$

$$= \lim_{+\infty} \frac{x\left(\sqrt{1-\frac{2}{x}}-2\right)}{x} = -1$$

$$\lim_{-\infty} \sqrt{x^2+3x-5}-2x = " +\infty + \infty " = +\infty$$

$$\lim_{-\infty} x^2+3x-5 = \lim_{-\infty} x^2 = +\infty \quad \text{donc}$$

$$\lim_{+\infty} \sqrt{x^2+3x-5}-2x = \lim_{+\infty} x\sqrt{1+\frac{3}{x}-\frac{5}{x^2}}-2x$$

$$= \lim_{+\infty} x\left(\sqrt{1+\frac{3}{x}-\frac{5}{x^2}}-2\right)$$

$$= " (+\infty)(-1) " = -\infty$$

Exercice 4-3

$$\lim_{+\infty} \sqrt{x^2 - 3x + 1} - x = \lim_{+\infty} x\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} - x$$

$$= \lim_{+\infty} x\left(\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} - 1\right)$$

$$= \lim_{+\infty} x\left(\frac{1 - \frac{3}{x} + \frac{1}{x^2} - 1}{\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} + 1}\right)$$

$$= \lim_{+\infty} \frac{-3 + 1/x}{\sqrt{1 - \frac{3}{x} + \frac{1}{x^2}} + 1} = \frac{-3}{2}$$

Exercice 4-4

$$\lim_{x \to +\infty} \sqrt{4x^2 - 2x + 1} - 2x = \lim_{+\infty} \sqrt{x^2 \cdot \left(4 - \frac{2}{x} + \frac{1}{x^2}\right)} - 2x$$

$$= \lim_{+\infty} x \cdot \sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} - 2x$$

$$= \lim_{+\infty} x \cdot \left(\sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} - 2\right)$$

$$= \lim_{+\infty} x \cdot \left(\frac{4 - 2/x + \frac{1}{x^2} - 4}{\sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} + 2}\right)$$

$$= \lim_{+\infty} \frac{-2 + 1/x}{\sqrt{4 - \frac{2}{x} + \frac{1}{x^2}} + 2} = -\frac{2}{4}$$

$$= -\frac{1}{2}$$

Exercice 4-5

$$\lim_{-\infty} \sqrt{x^2+3x+7}+x = \lim_{-\infty} -x\sqrt{1+\frac{3}{x}+\frac{7}{x^2}}+x$$

$$= \lim_{-\infty} -x\left(\sqrt{1+\frac{3}{x}+\frac{7}{x^2}}-1\right)$$

$$= \lim_{-\infty} -x\left(\frac{1+\frac{3}{x}+\frac{7}{x^2}-1}{\sqrt{1+\frac{3}{x}+\frac{7}{x^2}}+1}\right)$$

$$= \lim_{-\infty} \frac{-3-\frac{7}{x}}{\sqrt{1+\frac{3}{x}+\frac{7}{x^2}}+1} = \frac{-3}{2}$$

Exercice 4-6

$\lim\limits_{+\infty} x - 2\sqrt{x} + 9$

$= \lim\limits_{+\infty} x\left(1 - 2\frac{\sqrt{x}}{x} + \frac{9}{x}\right)$

$= \lim\limits_{+\infty} x\left(1 - 2\left(\frac{1}{\sqrt{x}}\right) + \frac{9}{x}\right) = +\infty$

$\lim\limits_{+\infty} \frac{\sqrt{x}}{x+1} = \lim\limits_{+\infty} \frac{x\left(\frac{\sqrt{x}}{x}\right)}{x\left(1+\frac{1}{x}\right)} = \frac{1/\sqrt{x}}{1+\frac{1}{x}} = \frac{0}{1}$

$= 0$

$\lim\limits_{+\infty} \frac{x - 2\sqrt{x} + 2}{x} = \lim\limits_{+\infty} 1 - 2\frac{\sqrt{x}}{x} + \frac{2}{x}$

$= \lim\limits_{+\infty} 1 - 2\left(\frac{1}{\sqrt{x}}\right) + \frac{2}{x} = 1$

$\lim\limits_{+\infty} \frac{x + \sqrt{x}}{x - \sqrt{x}} = \lim\limits_{+\infty} \frac{x\left(1 + \frac{\sqrt{x}}{x}\right)}{x\left(1 - \frac{\sqrt{x}}{x}\right)} = \lim\limits_{+\infty} \frac{1 + \frac{1}{\sqrt{x}}}{1 - \frac{1}{\sqrt{x}}} = 1$

$\lim\limits_{+\infty} \frac{x+2}{\sqrt{x}} = \lim\limits_{+\infty} \frac{x}{\sqrt{x}} + \frac{2}{\sqrt{x}}$

$= \lim\limits_{+\infty} \sqrt{x} + \frac{2}{\sqrt{x}} = +\infty$