Logique 3

EX 3-1

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \qquad n \in N^*$$

EX 3-2

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} \qquad n \in N^*$$

EX 3-3

$$(1 \times 2) + (2 \times 3) + \ldots + n(n+1) = \frac{1}{3}\, n(n+1)(n+2) \qquad n \in N^*$$

EX 3-4

$$\frac{1}{1 \times 2 \times 3} + \frac{1}{2 \times 3 \times 4} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)} \quad \forall n \in \mathbb{N}^*$$

EX 3-5

$$1 + q + q^2 + \ldots\ldots + q^n = \frac{1 - q^{n+1}}{1 - q} \qquad q \neq 1 \qquad \forall n \in \mathbb{N}^*$$

EX 3-6

$$1^3 + 3^3 + 5^3 + \ldots + (2n-1)^3 = n^2\left(2n^2 - 1\right) \qquad \forall n \in \mathbb{N}^*$$

EX 3-7

$$\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \ldots + \frac{1}{4n^2 - 1} = \frac{n}{2n+1} \qquad \forall n \in \mathbb{N}^*$$

EX 3-8

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1} n^2 = \frac{n(n+1)}{2} (-1)^{n+1} \qquad \forall n \in \mathbb{N}^*$$

EX 3-9

$A_n = 777\ldots\ldots7$ (n fois de 7) Montrons que : $A_n = \frac{7}{9}(10^n - 1) \quad \forall n \in \mathbb{N}^*$

EX 3-10

$2^n \times 1 \times 3 \times 5 \times \ldots \times (2n-1)) = (n+1)(n+2)\ldots\ldots(n+n) \qquad \forall n \in \mathbb{N}^*$