Solution exercice 42

ⓐ $V_n = U_n - 5 \Rightarrow V_{n+1} = U_{n+1} - 5$

$\Rightarrow V_{n+1} = \frac{2}{5}U_n + 3 - 5$

$\Rightarrow V_{n+1} = \frac{2}{5}U_n - 2$

$\Rightarrow V_{n+1} = \frac{2}{5}(U_n - 5) = \frac{2}{5}V_n$

$V_n)$ est géométrique de raison $q =$ et

de premier terme $V_0 = U_0 - 5 = \Rightarrow V_0 = -4$

ⓑ On a $V_n = V_0 \cdot q^n \Rightarrow V_n = -4 \cdot \left(\frac{2}{5}\right)^n$

on a : $V_n = U_n - 5 \Rightarrow U_n = V_n + 5$

$\Rightarrow U_n = -4\left(\frac{2}{5}\right)^n + 5$

$\lim\limits_{+\infty} \left(\frac{2}{5}\right)^n = 0$ car $-1 < \frac{2}{5} < 1$

donc $\lim\limits_{+\infty} U_n = \lim\limits_{+\infty} -4\cancel{\left(\frac{2}{5}\right)^n}^{\,0} + 5 = 5$

ⓒ $S_n = V_0 + V_1 + \ldots + V_n = V_0 \cdot \frac{1 - q^{n+1}}{1 - q}$

$= \left(\frac{V_0}{1-q}\right) \cdot \left(1 - q^{n+1}\right) = \left(\frac{-4}{1 - \frac{2}{5}}\right)\left(1 - \left(\frac{2}{5}\right)^{n+1}\right)$

$= \frac{-20}{3}\left(1 - \left(\frac{2}{5}\right)^{n+1}\right)$

$T_n = U_0 + U_1 + \ldots + U_n$ $\quad U_n = V_n + 5$

$= (V_0 + 5) + (V_1 + 5) + \cdots + (V_n + 5)$

$= (V_0 + V_1 + \cdots + V_n) + (5 + 5 + \cdots + 5)$

$= -\frac{20}{3}\left(1 - \left(\frac{2}{5}\right)^{n+1}\right) + 5(n+1)$